

CALCULUS INTEGRALS

DEFINITE INTEGRAL DEFINITION

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$$

where $\Delta x = \frac{b-a}{n}$ and $x_k = a + k\Delta x$

FUNDAMENTAL THEOREM OF CALCULUS

$$\int_a^b f(x) dx = [F(x)]_a^b = F(b) - F(a)$$

where f is continuous on $[a, b]$ and $F' = f$

INTEGRATION PROPERTIES

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx$$

$$\int_a^b f(x) \pm g(x) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

$$\int_a^a f(x) dx = 0 \text{ and } \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

COMMON INTEGRALS

$$\int k dx = kx + C$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C, n \neq -1$$

$$\int x^{-1} dx = \int \frac{1}{x} dx = \ln|x| + C$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| + C$$

$$\int \ln(x) dx = x \ln(x) - x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \frac{1}{a^2+u^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{u}{a}\right) + C$$

$$\int \frac{1}{\sqrt{a^2-u^2}} dx = \sin^{-1}\left(\frac{u}{a}\right) + C$$

APPROXIMATING DEFINITE INTEGRALS

Left-hand and right-hand rectangle approximations

$$L_n = \Delta x \sum_{k=0}^{n-1} f(x_k) \quad R_n = \Delta x \sum_{k=1}^n f(x_k)$$

Midpoint Rule

$$M_n = \Delta x \sum_{k=0}^{n-1} f\left(\frac{x_k + x_{k+1}}{2}\right)$$

Trapezoid Rule

$$T_n = \frac{\Delta x}{2} (f(x_0) + 2f(x_1) + 2f(x_2) + \dots + f(x_n))$$

TRIGONOMETRIC SUBSTITUTION

EXPRESSION	SUBSTITUTION	EXPRESSION EVALUATION	IDENTITY USED
$\sqrt{a^2 - x^2}$	$x = a \sin \theta$ $dx = a \cos \theta d\theta$	$\sqrt{a^2 - a^2 \sin^2 \theta}$ $= a \cos \theta$	$1 - \sin^2 \theta$ $= \cos^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta$ $dx = a \sec \theta \tan \theta d\theta$	$\sqrt{a^2 \sec^2 \theta - a^2}$ $= a \tan \theta$	$\sec^2 \theta - 1$ $= \tan^2 \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta$ $dx = a \sec^2 \theta d\theta$	$\sqrt{a^2 + a^2 \tan^2 \theta}$ $= a \sec \theta$	$1 + \tan^2 \theta$ $= \sec^2 \theta$

APPROXIMATION BY SIMPSON RULE FOR EVEN N

$$S_n = \frac{\Delta x}{3} (f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n))$$

INTEGRATION BY SUBSTITUTION

$$\int_a^b f(g(x)) g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

where $u = g(x)$ and $du = g'(x) dx$

COMMON INTEGRALS

$$\int \sec ax dx = \frac{1}{a} \ln|\sec ax + \tan ax| = \frac{1}{a} \ln\left|\tan\left(\frac{ax}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \csc ax dx = \frac{1}{a} \ln|\csc ax - \cot ax| = \frac{1}{a} \ln\left|\tan\left(\frac{ax}{2}\right)\right|$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$



INTEGRATION BY PARTS

$$\int u dv = uv - \int v du \quad \text{where } v = \int dv$$

or

$$\int f(x) g'(x) dx = f(x) g(x) - \int f'(x) g(x) dx$$