

مشتق های اصلی

$$\begin{aligned} \frac{d}{dx} (a) &= 0 \\ \frac{d}{dx} (ax^n) &= nax^{n-1} \\ \frac{d}{dx} (e^{ax}) &= ae^{ax} \\ \frac{d}{dx} (\sin ax) &= a \cos ax \\ \frac{d}{dx} (\cos ax) &= -a \sin ax \\ \frac{d}{dx} (\tan ax) &= a \sec^2 ax \\ \frac{d}{dx} (\cot ax) &= -a \csc^2 ax \\ \frac{d}{dx} (\sec x) &= \tan x \sec x \\ \frac{d}{dx} (\csc x) &= -\cot x \csc x \\ \frac{d}{dx} (\ln ax) &= \frac{1}{x} \end{aligned}$$

بسط های مهم

$$\begin{aligned} \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \quad [|x| < \infty] \\ \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \quad [|x| < \infty] \\ \tan x &= x + \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \dots \quad [|x| < \pi/2] \\ \arcsin x &= x + \frac{x^3}{2.3} + \frac{1.3}{2.4.5}x^5 + \frac{1.3.5}{2.4.6.7}x^7 + \dots \quad [|x| < 1] \\ \arccos x &= \frac{\pi}{2} - \arcsin x \quad [|x| < 1, 0 < \arccos x < \pi] \\ \arctan x &= x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots \quad [|x| < 1] \\ e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \quad [|x| < \infty] \\ \ln(1+x) &= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \quad [-1 < x \leq 1] \end{aligned}$$

انتگرال های پر کاربرد



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$$\begin{aligned} \int x^n dx &= \frac{x^{n+1}}{n+1} \quad (\text{provided } n \neq -1) \\ \int \frac{dx}{x} &= \int x^{-1} dx = \ln x \\ \int \frac{dx}{a+bx} &= \frac{1}{b} \ln(a+bx) \\ \int \frac{x dx}{a+bx} &= \frac{x}{b} - \frac{a}{b^2} \ln(a+bx) \\ \int \frac{dx}{x(x+a)} &= -\frac{1}{a} \ln \frac{x+a}{x} \\ \int \frac{dx}{(a+bx)^2} &= -\frac{1}{b(a+bx)} \\ \int \frac{dx}{a^2+x^2} &= \frac{1}{a} \tan^{-1} \frac{x}{a} \\ \int \frac{dx}{a^2-x^2} &= \frac{1}{2a} \ln \frac{a+x}{a-x} \quad (a^2-x^2 > 0) \\ \int \frac{dx}{x^2-a^2} &= \frac{1}{2a} \ln \frac{x-a}{x+a} \quad (x^2-a^2 > 0) \\ \int \frac{x dx}{a^2 \pm x^2} &= \pm \frac{1}{2} \ln(a^2 \pm x^2) \\ \int \frac{dx}{\sqrt{a^2-x^2}} &= \sin^{-1} \frac{x}{a} = -\cos^{-1} \frac{x}{a} \quad (a^2-x^2 > 0) \\ \int \frac{dx}{\sqrt{x^2 \pm a^2}} &= \ln(x + \sqrt{x^2 \pm a^2}) \\ \int \frac{x dx}{\sqrt{a^2-x^2}} &= -\sqrt{a^2-x^2} \\ \int \frac{x dx}{\sqrt{x^2 \pm a^2}} &= \sqrt{x^2 \pm a^2} \\ \int \sqrt{a^2-x^2} dx &= \frac{1}{2} \left(x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right) \\ \int x\sqrt{a^2-x^2} dx &= -\frac{1}{3} (a^2-x^2)^{3/2} \\ \int \sqrt{x^2 \pm a^2} dx &= \frac{1}{2} [x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2})] \\ \int x(\sqrt{x^2 \pm a^2}) dx &= \frac{1}{3} (x^2 \pm a^2)^{3/2} \\ \int e^{ax} dx &= \frac{1}{a} e^{ax} \\ \int \ln ax dx &= (x \ln ax) - x \\ \int xe^{ax} dx &= \frac{e^{ax}}{a^2} (ax - 1) \\ \int \frac{dx}{a+be^{cx}} &= \frac{x}{a} - \frac{1}{ac} \ln(a+be^{cx}) \\ \int \sin ax dx &= -\frac{1}{a} \cos ax \\ \int \cos ax dx &= \frac{1}{a} \sin ax \\ \int \tan ax dx &= \frac{1}{a} \ln(\cos ax) = \frac{1}{a} \ln(\sec ax) \\ \int \cot ax dx &= \frac{1}{a} \ln(\sin ax) \\ \int \sec ax dx &= \frac{1}{a} \ln(\sec ax + \tan ax) = \frac{1}{a} \ln \left[\tan \left(\frac{ax}{2} + \frac{\pi}{4} \right) \right] \\ \int \csc ax dx &= \frac{1}{a} \ln(\csc ax - \cot ax) = \frac{1}{a} \ln \left(\tan \frac{ax}{2} \right) \\ \int \sin^2 ax dx &= \frac{x}{2} - \frac{\sin 2ax}{4a} \\ \int \cos^2 ax dx &= \frac{x}{2} + \frac{\sin 2ax}{4a} \\ \int \frac{dx}{\sin^2 ax} &= -\frac{1}{a} \cot ax \\ \int \frac{dx}{\cos^2 ax} &= \frac{1}{a} \tan ax \\ \int \tan^2 ax dx &= \frac{1}{a} (\tan ax) - x \\ \int \cot^2 ax dx &= -\frac{1}{a} (\cot ax) - x \\ \int \sin^{-1} ax dx &= x(\sin^{-1} ax) + \frac{\sqrt{1-a^2x^2}}{a} \\ \int \cos^{-1} ax dx &= x(\cos^{-1} ax) - \frac{\sqrt{1-a^2x^2}}{a} \\ \int \frac{dx}{(x^2+a^2)^{3/2}} &= \frac{x}{a^2\sqrt{x^2+a^2}} \\ \int \frac{x dx}{(x^2+a^2)^{3/2}} &= -\frac{1}{\sqrt{x^2+a^2}} \end{aligned}$$