

**Right triangle definition**

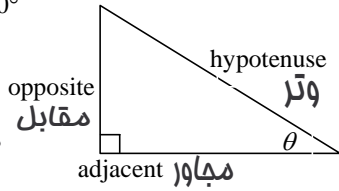
$0 < \theta < \frac{\pi}{2}$  or  $0^\circ < \theta < 90^\circ$

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

**تعریف روابط در مثلث**



**رسم توابع اصلی**

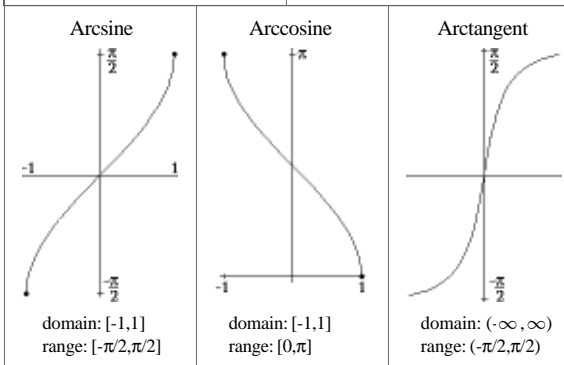
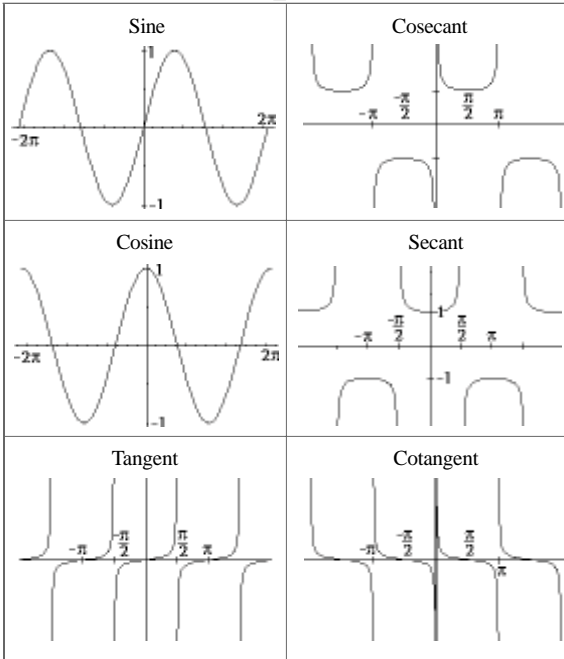
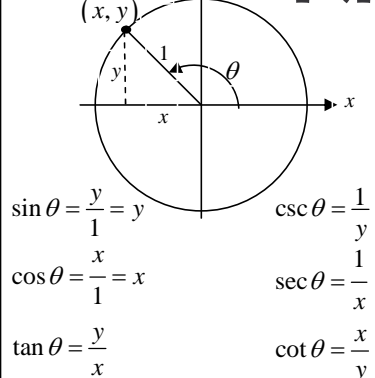
**Domain دامنه توابع مثلثاتی**

- $\sin \theta$ ,  $\theta$  can be any angle
- $\cos \theta$ ,  $\theta$  can be any angle
- $\tan \theta$ ,  $\theta \neq \left(n + \frac{1}{2}\right)\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$
- $\csc \theta$ ,  $\theta \neq n\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$
- $\sec \theta$ ,  $\theta \neq \left(n + \frac{1}{2}\right)\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$
- $\cot \theta$ ,  $\theta \neq n\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

**Range برد**

$-1 \leq \sin \theta \leq 1$      $-1 \leq \cos \theta \leq 1$   
 $-\infty \leq \tan \theta \leq \infty$      $-\infty \leq \cot \theta \leq \infty$   
 $\csc \theta \geq 1$  and  $\csc \theta \leq -1$   
 $\sec \theta \geq 1$  and  $\sec \theta \leq -1$

**Unit circle دایره یکه**



**Tangent and Cotangent Identities**

شناسایی تانژانت - کتانژانت - سکانت - کسکانت

$\tan \theta = \frac{\sin \theta}{\cos \theta}$      $\cot \theta = \frac{\cos \theta}{\sin \theta}$   
 $\csc \theta = \frac{1}{\sin \theta}$      $\sin \theta = \frac{1}{\csc \theta}$   
 $\sec \theta = \frac{1}{\cos \theta}$      $\cos \theta = \frac{1}{\sec \theta}$   
 $\cot \theta = \frac{1}{\tan \theta}$      $\tan \theta = \frac{1}{\cot \theta}$

**Pythagorean Identities روابط فیثاغورثی**

$\sin^2 \theta + \cos^2 \theta = 1$   
 $\tan^2 \theta + 1 = \sec^2 \theta$   
 $1 + \cot^2 \theta = \csc^2 \theta$

**Even/Odd Formulas فرمول های زوج و فرد**

$\sin(-\theta) = -\sin \theta$      $\csc(-\theta) = -\csc \theta$   
 $\cos(-\theta) = \cos \theta$      $\sec(-\theta) = \sec \theta$   
 $\tan(-\theta) = -\tan \theta$      $\cot(-\theta) = -\cot \theta$

**Periodic Formulas فرمول های تناوب**

$\sin(\theta + 2\pi n) = \sin \theta$      $\csc(\theta + 2\pi n) = \csc \theta$   
 $\cos(\theta + 2\pi n) = \cos \theta$      $\sec(\theta + 2\pi n) = \sec \theta$   
 $\tan(\theta + \pi n) = \tan \theta$      $\cot(\theta + \pi n) = \cot \theta$

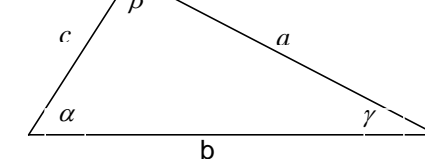
**Double Angle Formulas**

فرمول های دو برابر کمان

$\sin(2\theta) = 2 \sin \theta \cos \theta$   
 $\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$   
 $\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$

**Law of Sines, Cosines**

قانون سینوس ها - کسینوس ها



**Law of Cosines**

قانون کسینوس ها  
 $a^2 = b^2 + c^2 - 2bc \cos \alpha$   
 $b^2 = a^2 + c^2 - 2ac \cos \beta$   
 $c^2 = a^2 + b^2 - 2ab \cos \gamma$

**Law of Sines**

قانون سینوس ها  
 $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$

**Half Angle Formulas**

فرمول های نصف کمان

$\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$   
 $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$   
 $\tan^2 \theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

**Sum and Difference Formulas**

فرمول های جمع و تفریق

$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$   
 $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$   
 $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$

**Product to Sum Formulas**

تبدیل ضرب به جمع

$\sin \alpha \sin \beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$   
 $\cos \alpha \cos \beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$   
 $\sin \alpha \cos \beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$   
 $\cos \alpha \sin \beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

**Sum to Product Formulas**

تبدیل جمع به ضرب

$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$   
 $\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$   
 $\cos \alpha + \cos \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$   
 $\cos \alpha - \cos \beta = -2 \sin\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$

**Cofunction Formulas**

فرمول های متمم

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos \theta$      $\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$   
 $\csc\left(\frac{\pi}{2} - \theta\right) = \sec \theta$      $\sec\left(\frac{\pi}{2} - \theta\right) = \csc \theta$   
 $\tan\left(\frac{\pi}{2} - \theta\right) = \cot \theta$      $\cot\left(\frac{\pi}{2} - \theta\right) = \tan \theta$